# REALIZATION OF HIGH PRESSURES AND TEMPERATURES IN THE GAS PHASE OF A BUBBLE LIQUID FLOWING THROUGH A NOZZLE 

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A steady flow of a bubble gas-liquid mixture in a nozzle having a circular cross section has been investigated. The possibility of realization of superhigh temperatures and pressures in the gas phase of the mixture in the region near the smallest cross section of the nozzle has been analyzed. The influence of the initial radius of the flow and the volume content of bubbles, determining the volumetric rate of flow of the liquid supplied to the nozzle, on the pattern of the flow has been considered.

Introduction. At present, great interest is being shown in the problem on obtaining of high pressures and temperatures in the gas phase of bubble liquids. An efficient method of solving this problem, tested in practice, is excitation of vibrations of bubbles or a cluster of bubbles by a pressure pulse [1]. Galimov et al. of [2], on the basis of an experimental investigation of a benzene flow in a nozzle, occurring with a cavitation, concluded that diamondlike systems can be obtained under superhigh pressures created in bubbles.

Consequently, the study of the mechanisms of formation of such flows of vapor-gas-liquid mixtures, in which the gas in the bubbles is subjected to extremum pressures and temperatures, can offer the prospect, unlike the methods of pulse action on "immovable" bubbles, of realization of continuous technological processes under superhigh pressures and temperatures.

Formulation of the Problem and Basic Equations. We consider a flow of a monodisperse bubble gas-liquid mixture in a nozzle, for which the initial parameters and the flow velocity at the inlet to the nozzle can be prescribed. A mathematical model of the indicated flow is constructed on the assumption that, in each elementary volume, all the bubbles are spherical and have equal radii, the rates of motion of the phases are equal, the viscosity and heat conduction are significant only in the interphase interaction, in particular, in the process of bubble pulsations, phase transitions are absent, and bubbles do not coalesce and do not break. The friction of the flow on the walls of the channel is disregarded.

In line with these assumptions, we write the system of macroscopic equations for the mass of the liquid, the number of bubbles, and the pulses and pressures in the bubbles in a quasi-steady flow of the mixture being considered in the quasi-one-dimensional approximation

$$
\begin{gather*}
\frac{d}{d z}\left[\rho_{\text {liq }}^{0}\left(1-\alpha_{\mathrm{g}}\right) v S\right]=0, \frac{d}{d z}(n v S)=0, \\
\rho_{\mathrm{liq}}^{0}\left(1-\alpha_{\mathrm{g}}\right) v \frac{d v}{d z}=-\frac{d p_{\text {liq }}}{d z}, v \frac{d p_{\mathrm{g}}}{d z}=-\frac{3 \gamma p_{\mathrm{g}}}{a} w-\frac{3(\gamma-1)}{a} q,  \tag{1}\\
\alpha_{\mathrm{liq}}+\alpha_{\mathrm{g}}=1, \quad \alpha_{\mathrm{g}}=\frac{4}{3} \pi a^{3} n .
\end{gather*}
$$

Assuming that the radial motion of bubbles proceeds in accordance with the Rayleigh-Lamb equation, we obtain

[^0]\[

$$
\begin{equation*}
a v \frac{d w}{d z}+\frac{3}{2} w^{2}+4 v^{(v)} \frac{w}{a}=\frac{p_{\mathrm{g}}-p_{\mathrm{liq}}-\frac{2 \sigma}{a}}{\rho_{\mathrm{liq}}^{0}}, v \frac{d a}{d z}=w . \tag{2}
\end{equation*}
$$

\]

It is assumed that the liquid is incompressible and the gas is calorically perfect: $\rho_{\text {liq }}^{0}=$ const; $p_{\mathrm{g}}=\rho_{\mathrm{g}}^{0} R T_{\mathrm{g}}$. The heat flow $q$ is defined by the approximate finite relation [3]

$$
q=\mathrm{Nu} \lambda_{\mathrm{g}} \frac{T_{\mathrm{g}}-T_{0}}{2 a}, \quad \mathrm{Nu}=\left\{\begin{array}{ll}
\sqrt{\mathrm{Pe}}, & \mathrm{Pe} \geq 100, \\
10, & \mathrm{Pe}<100,
\end{array} \quad \mathrm{Pe}=12(\gamma-1) \frac{T_{0}}{\left|T_{\mathrm{g}}-T_{0}\right|} \frac{a|w|}{v^{(T)}}, \quad v^{(T)}=\frac{\lambda_{\mathrm{g}}}{c_{\mathrm{g}} \rho_{\mathrm{g}}^{0}} .\right.
$$

Using the equations of state and continuity for the temperature of the gas in the bubbles, we obtain

$$
\frac{T_{\mathrm{g}}}{T_{0}}=\frac{p_{\mathrm{g}}}{p_{\mathrm{g} 0}}\left(\frac{a}{a_{0}}\right)^{3} .
$$

Method of Numerical Calculation. For numerical analysis of the problem on a steady bubble-liquid flow in a nozzle, we rearrange Eqs. (1) and (2) as

$$
\begin{gathered}
\frac{d v}{d z}=\frac{3 \alpha_{\mathrm{g}} w}{a}-\frac{v}{S} \frac{d S}{d z}, \frac{d p_{\mathrm{liq}}}{d z}=\rho_{\mathrm{liq}}^{0}\left(\alpha_{\mathrm{g}}-1\right) v\left(\frac{3 \alpha_{\mathrm{g}} w}{a}-\frac{v}{S} \frac{d S}{d z}\right), \frac{d p_{\mathrm{g}}}{d z}=-\frac{1}{a v}\left(3 \gamma p_{\mathrm{g}} w+3(\gamma-1) q\right), \\
\frac{d w}{d z}=\frac{1}{a v}\left(\frac{p_{\mathrm{g}}-p_{\mathrm{liq}}-\frac{2 \sigma}{a}}{\rho_{\mathrm{liq}}^{0}}-\frac{3}{2} w^{2}-4 v^{(v) w} \frac{w}{a}\right), \frac{d a}{d z}=\frac{w}{v} .
\end{gathered}
$$

Using the relation between the number of bubbles and their concentration

$$
n=\frac{3}{4} \frac{\alpha_{\mathrm{g}}}{\pi d^{3}}
$$

we write the following relation for the volume content of the gas phase in an arbitrary cross section of the nozzle:

$$
\alpha_{\mathrm{g}}=\alpha_{\mathrm{g} 0}\left(\frac{a}{a_{0}}\right)^{3} \frac{v_{0} S_{0}}{v S} .
$$

Results of Calculations. In a bubble liquid flowing through a nozzle there can arise nonlinear vibrations of bubbles because of the decrease in the pressure in the narrowing part of the nozzle and the inertial radial motion of bubbles in the region of the smallest cross section. The intensity of these vibrations is determined by the characteristics of the gas-liquid mixture and the minimum pressure attained at the neck of the nozzle.

A decrease in the pressure of the liquid in the narrowing part of the nozzle causes the bubbles to grow. The pressure of the liquid phase at the neck of the nozzle reaches a minimum value and begins to increase in the broadening part of the nozzle. In this case, the gas bubbles, growing in the narrowing part of the nozzle, pass through the equilibrium state at the smallest cross section and continue to grow because of the inertial motion of their walls and reach maximum sizes in the broadening part of the nozzle near the neck. Under the action of the increasing pressure in the liquid, the bubbles in the broadening region collapse abruptly. In the process of collapse, bubbles pass again through the equilibrium state and, in doing so, decrease to minimum sizes, which lead to a large increase in the pressure of the gas in the bubbles and a new growth of them due to the difference between the pressures of the gas in the bubbles and in the liquid at the instant the compression of the bubbles is maximum. Then these nonlinear vibrations of bubbles in the broadening part of the nozzle damp gradually due to the dissipation arising as a result of the heat exchange and the viscosity of the liquid. In this case, as the calculations have shown, the average pressure and temperature of the gas in the bubbles in the process of their collapse reach values that are much larger than the initial ones.


Fig. 1. Profile of a nozzle.


Fig. 2. Calculated distributions of the dimensionless radius of the bubbles and the gas pressure and temperature along the length of the nozzle: a) $a_{0}=$ $\left.9 \cdot 10^{-5} \mathrm{~m}, \alpha_{\mathrm{g} 0}=10^{-4}, v_{0}=11.46 \mathrm{~m} / \mathrm{sec}, p_{\mathrm{g}}^{\mathrm{m}}=32 \mathrm{MPa}, T_{\mathrm{g}}^{\mathrm{m}}=2900 \mathrm{~K} ; \mathrm{b}\right)$ $a_{0}=2 \cdot 10^{-4} \mathrm{~m}, \alpha_{\mathrm{g} 0}=2 \cdot 10^{-4}, v_{0}=11.45 \mathrm{~m} / \mathrm{sec}, p_{\mathrm{g}}^{\mathrm{m}}=200 \mathrm{MPa}, T_{\mathrm{g}}^{\mathrm{m}}=5900$ K ; c) $a_{0}=3 \cdot 10^{-5} \mathrm{~m}, \alpha_{\mathrm{g} 0}=10^{-4}, v_{0}=11.45 \mathrm{~m} / \mathrm{sec}$.

The numerical investigation was carried out for a nozzle (Fig. 1) having a variable circular cross section of length 0.15 m . The diameters of the ends are equal to $3.57 \cdot 10^{-2} \mathrm{~m}$. The smallest cross section is at a distance of $2.3 \cdot 10^{-2} \mathrm{~m}$ from the inlet to the nozzle, where $z=0$, and its diameter is equal to $2.52 \cdot 10^{-2} \mathrm{~m}$.

In the calculations, water was used as the liquid and air was used as the gas phase. We considered a bubbleliquid flow, in which the bubbles had an initial radius $a_{0}$ falling within the range $3 \cdot 10^{-5}-2 \cdot 10^{-4} \mathrm{~m}$, and the initial volume gas content $\alpha_{g 0}$ ranged from $5 \cdot 10^{-5}$ to $2 \cdot 10^{-4}$. At the inlet to the nozzle, the pressure of the phases and the velocity of the liquid flow were prescribed. In all cases, the temperature of the liquid was assumed to be equal to $T_{0}=300 \mathrm{~K}$.

A liquid flow with strong nonlinear vibrations of bubbles is realized when the pressure at the neck of the nozzle decreases to several thousandths of an atmosphere and is determined by the velocity of the flow at the inlet to the nozzle and the ratio between the area of the input cross and the area of the cross section at the neck of the nozzle. An increase in the velocity of the flow to the limiting value, at which the pressure of the liquid at the neck tends to zero, gives rise to the largest vibrations of the bubbles and causes the pressure and temperature of the gas in the bubbles to increase to enormous values at the instants they are compressed to a maximum degree. For example, at the instants the bubbles of radius $a_{0}=9 \cdot 10^{-5} \mathrm{~m}$ in a gas-liquid mixture with a volume gas content $\alpha_{\mathrm{g} 0}=10^{-4}$ are compressed to a maximum degree at pressures of the phases $p_{\mathrm{liq} 0}=p_{\mathrm{g} 0}=0.2 \mathrm{MPa}$ and a liquid-flow velocity $v_{0}=$
$11.46 \mathrm{~m} / \mathrm{sec}$ at the inlet to the nozzle, the pressure and temperature of the gas in the bubbles reach $p_{\mathrm{g}}^{\mathrm{m}}=32 \mathrm{MPa}$ and $T_{\mathrm{g}}^{\mathrm{m}}=2900 \mathrm{~K}$ (Fig. 2a). The dashed curve corresponds to the pressure in the liquid and the solid line corresponds to the pressure of the gas in the bubbles.

When a bubble-liquid flow at the inlet to the nozzle has a maximum velocity at which the pressure of the liquid at the smallest cross section of the nozzle is close to zero, the highest pressures and temperatures of the gas phase (exceeding 100 MPa ) are realized in larger bubble systems ( $a_{0} \geq 10^{-4} \mathrm{~m}$ ). This is explained by the fact that, in this case, the specific surface through which the gas in the bubbles and the liquid exchange heat is smaller and the bubbles in the compression stage behave practically adiabatically (Fig. 2b).

In liquids with bubbles having smaller initial radii $\left(a_{0} \leq 5 \cdot 10^{-5} \mathrm{~m}\right)$, the vibrations of bubbles are weaker and superhigh pressures and temperatures are practically not realized in them because the surface forces of these liquid are stronger (Fig. 2c).

Our calculations have shown that the characteristic zone of vibrations of bubbles in a nozzle is determined by their initial sizes, and this zone broadens with increase in the volume content of the gas phase in a mixture. Note that the values presented for the pressure and temperature distributions along the nozzle correspond to the values averaged over a bubble. What actually happens is that the process of inertial compression of each individual bubble represents a motion accompanied by nonlinear gas-dynamic effects, such as formation of shock waves. Therefore, in the central zones of the bubbles, much higher temperatures and pressures can arise at certain instants of time, as compared to the temperatures and pressures obtained by us.

Conclusions. The numerical analysis performed by us for a flow of a bubble gas-liquid mixture in a nozzle has shown that superhigh pressures and temperatures can be obtained in the gas phase of an initially "cold" bubble system as a result of the initiation of intense nonlinear vibrations of bubbles in the broadening part of the nozzle near its smallest cross section. It has been established that, in the case where a liquid flow has a maximum velocity at the inlet to the nozzle, the highest temperatures and pressures are realized in the gas of the bubbles with a radius exceeding $10^{-4} \mathrm{~m}$ because of their practically adiabatic behavior in the process of pulsations. The calculation data presented allow the conclusion that the bubble liquid-nozzle system can be used for realization of continuous technological processes occurring with superhigh temperatures and pressures.

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## NOTATION

$a$, radius of a bubble, $\mathrm{m} ; c_{\mathrm{g}}$, specific heat of the gas, $\mathrm{J} /(\mathrm{kg} \cdot \mathrm{K}) ; n$, number of bubbles in a unit volume, $\mathrm{m}^{-3} ; \mathrm{Nu}$, Nusselt number; $p_{i}$, pressure of phases, $\mathrm{Pa} ; \mathrm{Pe}$, Peclet number; $q$, intensity of the heat flow between the liquid and the gas in a bubble referred to a unit area of the bubble surface, $\mathrm{W} / \mathrm{m}^{2} ; R$, universal gas constant, $\mathrm{J} /($ mole $\cdot \mathrm{K}) ; R_{\mathrm{n}}$, radius of a nozzle, $\mathrm{m} ; S$, area of a cross section of the nozzle, $\mathrm{m}^{2} ; T_{0}=$ const, temperature of the liquid, $\mathrm{K} ; T_{\mathrm{g}}$, temperature of the gas, $\mathrm{K} ; v$, velocity of a liquid flow, $\mathrm{m} / \mathrm{sec} ; w$, radial velocity of bubbles, $\mathrm{m} / \mathrm{sec} ; z$, spatial coordinate, $\mathrm{m} ; \alpha_{i}$, volume content of phases (dimensionless); $\gamma$, adiabatic index for the gas; $\lambda_{\mathrm{g}}$, heat conductivity of the gas, $\mathrm{W} /(\mathrm{m} \cdot \mathrm{K}) ; \mathrm{v}^{(v)}$, kinematic viscosity of the liquid, $\mathrm{m}^{2} / \mathrm{sec} ; \mathrm{v}^{(T)}$, thermal diffusivity of the $\mathrm{gas}, \mathrm{m}^{2} / \mathrm{sec}$; $\rho_{i}^{0}$, true densities of the phases, $\mathrm{kg} / \mathrm{m}^{3} ; \sigma$, surface-tension coefficient, $\mathrm{N} / \mathrm{m}$. Subscripts: 0 (overhead), true value of a parameter; 0 (beneath), value of a parameter at the inlet of the nozzle; m, maximum value of a parameter; n, nozzle; liq, liquid; g, gas.

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